11. Three charges are arranged as shown in Figure P15.11. Find the magnitude and direction of the electrostatic force on the charge at the origin.

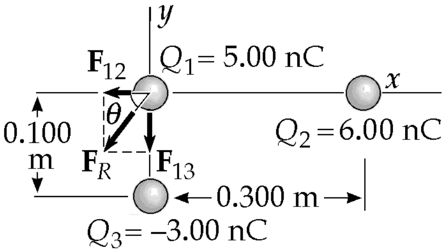

Figure P15.11

Solution The strategy to use when calculating the force exerted by a set of several point charges is to first find the force exerted by each of the point charges individually. The force exerted by the entire set of charges is the resultant of the forces exerted by the individual charges.

The charge Q_2 exerts a repulsive force, $\mathbf{F}_{12}$ on Q_1 as shown in the sketch. The distance separating these charges is $r_{12} = 0.300\text{ m}$ so the magnitude $F_{12} = k_e|Q_1||Q_2|/r_{12}^2$ becomes

$$F_{12} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(5.00\times10^{-9}\text{ C}\right)\left(6.00\times10^{-9}\text{ C}\right)}{(0.300\text{ m})^2} = 3.00\times10^{-6}\text{ N}$$

The negative charge Q_3 exerts an attractive force, $\mathbf{F}_{13}$ on Q_1. The distance between Q_1 and Q_3 is $r_{13} = 0.100\text{ m}$, so $F_{13} = k_e|Q_1||Q_3|/r_{13}^2$ becomes

$$F_{13} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(5.00\times10^{-9}\text{ C}\right)\left(3.00\times10^{-9}\text{ C}\right)}{(0.100\text{ m})^2} = 1.35\times10^{-5}\text{ N}$$

The magnitude of the resultant of $\mathbf{F}_{12}$ and $\mathbf{F}_{13}$ can be found by using the Pythagorean theorem as

$$F_R = \sqrt{(F_{12})^2 + (F_{13})^2} = \sqrt{\left(3.00\times10^{-6}\text{ N}\right)^2 + \left(1.35\times10^{-5}\text{ N}\right)^2} = 1.38\times10^{-5}\text{ N}$$

The angle that F_R makes with the horizontal is

$$\theta = \tan^{-1}\left(\frac{F_{13}}{F_{12}}\right) = \tan^{-1}\left(\frac{1.35\times10^{-5}\text{ N}}{3.00\times10^{-6}\text{ N}}\right) = 77.5^\circ$$

Thus, the resultant force exerted on the charge at the origin is

$$\mathbf{F}_R = 1.38\times10^{-5}\text{ N at }77.5^\circ\text{ below the negative }x\text{-axis}$$ ◊