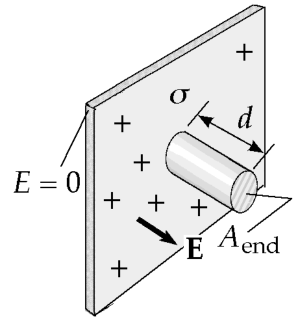
45. An infinite plane conductor has charge spread out on its surface as shown in Figure P15.45. Use Gauss's law to show that the electric field at any point outside the conductor is given by $E = \sigma / \epsilon_0$, where σ is the charge per unit area on the conductor. (**Hint:** Choose a gaussian surface in the shape of a cylinder with one end inside the conductor and one end outside the conductor.)



Solution At equilibrium, all excess charge on a conductor resides on the surface. For an infinite plane conductor (shown in an edge-on view in the figure), a uniform charge per unit area σ exists on each surface of the conductor. At points just outside a conducting surface, the electric field is perpendicular to the surface, and the field is zero at all points inside a conductor.

Gauss's law states:

> "The net electric flux through any closed gaussian surface is equal to the net charge enclosed by that surface divided by ϵ_0 ".

This may be used to determine the electric field strength just outside the surface of the plane conductor. Consider the cylindrical gaussian surface shown. The cylinder is oriented with its axis perpendicular to the plane conductor. One end of the cylinder is inside the conducting material and the other end is a distance d outside the conductor.

The flux passing through any part of a gaussian surface is given by $\Phi_E = EA\cos\theta$ where A is the area of that part of the surface, E is the field strength at that point on the surface, and θ is the angle between the direction of the field and the line perpendicular to the surface at that point.

The net flux through the entire gaussian surface may be written as

$$(\Phi_E)_{\text{net}} = (\Phi_E)_{\text{left end}} + (\Phi_E)_{\text{cylindrical side}} + (\Phi_E)_{\text{right end}}$$

Since the electric field is zero everywhere inside the conductor, $(\Phi_E)_{\text{left end}} = 0$.

Outside the conductor, the field is parallel to the side of the cylinder, so that the angle between the field and a line perpendicular to the cylindrical side is 90°.

Hence, $(\Phi_E)_{\text{cylindrical side}} = EA_{\text{side}}\cos 90° = 0$

The field is perpendicular to the right end; thus, $\theta = 0°$ here,

and $(\Phi_E)_{\text{right end}} = EA_{\text{end}}\cos 0° = EA_{\text{end}}$.

The charge enclosed by the gaussian surface is the charge on the circular area of the plane that lies inside the cylinder. This area equals the cross-sectional area of the cylinder and is the same as the area of the end, A_{end}.

Therefore, $Q_{\text{enclosed}} = \sigma A_{\text{end}}$

and Gauss's law $(\Phi_E)_{\text{net}} = Q_{\text{enclosed}}/\epsilon_0$ gives $0 + 0 + EA_{\text{end}} = \dfrac{\sigma A_{\text{end}}}{\epsilon_0}$

Thus, outside the infinite plane, $E = \sigma / \epsilon_0$ and is perpendicular to the plane. ◊

This result is observed to be the sum of the electric fields due to two parallel, infinite sheets of charge. The reason for this is that the conducting plate has a sheet of charge, of charge density σ, on each of its two surfaces.