19. In Rutherford's famous scattering experiments (which led to the planetary model of the atom), alpha particles (having charges of $+2e$ and masses of 6.64×10^{-27} kg) were fired toward a gold nucleus with charge $+79e$. An alpha particle, initially very far from the gold nucleus, is fired at 2.00×10^7 m/s directly toward the gold nucleus as in Figure P16.19. How close does the alpha particle get to the gold nucleus before turning around? Assume the gold nucleus remains stationary.

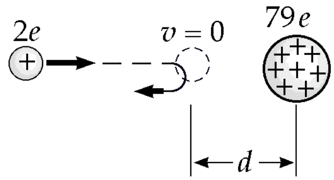

Figure P16.19

Solution As the alpha particle approaches the gold nucleus, the only force acting on it is the electrical force exerted by the nucleus. This force is not constant so we cannot easily use $F = ma$. However, this is a conservative force and the total mechanical energy of the system remains constant:

$$(KE_f + PE_f = KE_i + PE_i)$$

Since the gold nucleus is "fixed" (i.e., is so massive in comparison to the alpha particle that its recoil may be ignored), the total kinetic energy is that of the alpha particle,

$$KE = \frac{1}{2} m_\alpha v_\alpha^2$$

The potential energy of this pair of charges is $PE = k_e Q_1 Q_2 / r$, and the conservation of energy equation becomes

$$\frac{1}{2} m_\alpha v_f^2 + \frac{k_e Q_1 Q_2}{r_f} = \frac{1}{2} m_\alpha v_i^2 + \frac{k_e Q_1 Q_2}{r_i}$$

When the alpha particle is the minimum distance from the gold nucleus, $r_f = d$ and $v_f = 0$. Also, the alpha particle is initially "very far" from the nucleus, so $r_i \approx \infty$ and $v_i = 2.00 \times 10^7$ m/s. The energy equation is then

$$0 + \frac{k_e Q_1 Q_2}{d} = \frac{1}{2} m_\alpha v_i^2 + 0$$

and the closest the alpha particle gets to the nucleus is $d = 2k_e Q_1 Q_2 / m_\alpha v_i^2$:

$$d = \frac{2k_e(2e)(79e)}{m_\alpha v_i^2} = \frac{316\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{\left(6.64 \times 10^{-27} \text{ kg}\right)\left(2.00 \times 10^7 \text{ m/s}\right)^2} = 2.74 \times 10^{-14} \text{ m} \quad \Diamond$$