

37. A $25.0\text{-}\mu\text{F}$ capacitor and a $40.0\text{-}\mu\text{F}$ capacitor are charged by being connected across separate 50.0-V batteries. (a) Determine the resulting charge on each capacitor. (b) The capacitors are then disconnected from their batteries and connected to each other, with each negative plate connected to the other positive plate. What is the final charge of each capacitor, and what is the final potential difference across the $40.0\text{-}\mu\text{F}$ capacitor?

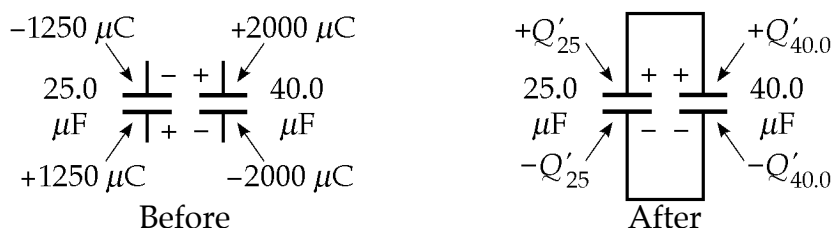
Solution

(a) The charge stored in a capacitor of capacitance C when a potential difference ΔV is maintained between its plates is $Q = C(\Delta V)$. Thus, when the given capacitors are each connected across a 50.0-V battery, the charges stored are

$$Q_{25} = (25.0 \mu\text{F})(50.0 \text{ V}) = 1.25 \times 10^3 \mu\text{C}$$

and $Q_{40} = (40.0 \mu\text{F})(50.0 \text{ V}) = 2.00 \times 10^3 \mu\text{C}$ ◇

(b) When the capacitors are carefully disconnected from the batteries, the capacitors are charged as shown in the “Before” figure below.



When the capacitors are reconnected, with the positive plate of one connected to the negative plate of the other, total charge is conserved but redistributed between the two capacitors. The net positive charge that existed on the two upper plates ($+2000 \mu\text{C} - 1250 \mu\text{C} = +750 \mu\text{C}$) is shared by these two plates as shown in the “After” figure. Likewise, the net negative charge ($-750 \mu\text{C}$) that originally existed on the two lower plates is shared by those plates. Thus, after the capacitors are reconnected,

$$Q'_{40} + Q'_{25} = 750 \mu\text{C} \quad [1]$$

Also, after the capacitors are reconnected, they are in parallel and have the same potential difference across them.

That is, $\Delta V'_{40} = \Delta V'_{25}$, or
$$\frac{Q'_{40}}{40.0 \mu\text{F}} = \frac{Q'_{25}}{25.0 \mu\text{F}}$$

which becomes
$$Q'_{40} = 1.60 Q'_{25} \quad [2]$$

Substituting Equation [2] into [1],
$$2.60 Q'_{25} = 750 \mu\text{C}$$

Thus, the final charge on the $25.0\text{-}\mu\text{F}$ capacitor is

$$Q'_{25} = 288 \mu\text{C} \quad \diamond$$

Then, Equation [1] gives:
$$Q'_{40} = 750 \mu\text{C} - 288 \mu\text{C} = 462 \mu\text{C}$$

The final potential difference across each capacitor is:

$$\Delta V'_{40} = \Delta V'_{25} = \frac{Q'_{25}}{25.0 \mu\text{F}} = \frac{288 \mu\text{C}}{25.0 \mu\text{F}} = 11.5 \text{ V} \quad \diamond$$