53. Three parallel-plate capacitors are constructed, each having the same plate area A, and with C_1 having plate spacing d_1, C_2 having plate spacing d_2, and C_3 having plate spacing d_3. Show that the total capacitance C of these three capacitors connected in series is the same as a capacitor of plate area A and with plate spacing $d = d_1 + d_2 + d_3$.

Solution

The capacitance of a parallel-plate capacitor, with vacuum between the plates, is $C = \epsilon_0 A/d$, where ϵ_0 is a constant, A is the area of one of the plates, and d is the distance between the plates. When three capacitors (having capacitances of C_1, C_2, and C_3) are connected in series, the total capacitance of the combination is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

If the capacitors all have the same plate area A, and plate spacings of d_1, d_2, and d_3 respectively, this becomes

$$\frac{1}{C_{eq}} = \frac{d_1}{\epsilon_0 A} + \frac{d_2}{\epsilon_0 A} + \frac{d_3}{\epsilon_0 A} = \frac{d_1 + d_2 + d_3}{\epsilon_0 A} \qquad \text{or} \qquad C_{eq} = \frac{\epsilon_0 A}{d_1 + d_2 + d_3}$$

Comparing this result to the general expression for the capacitance of a parallel-plate capacitor, $C = \epsilon_0 A/d$, it is observed that the total capacitance of the series combination is the same as that of a single capacitor of plate area A and plate spacing $d = d_1 + d_2 + d_3$. ◊