8. An aluminum wire with a cross-sectional area of $4.0\times10^{-6}\ \text{m}^2$ carries a current of 5.0 A. Find the drift speed of the electrons in the wire. The density of aluminum is $2.7\ \text{g}/\text{cm}^3$. (Assume that one electron is supplied by each atom.)

Solution

In terms of the speed v_d of the drifting electrons, the current in a metallic conductor is $I = neAv_d$. Here, n is the number of free electrons per unit volume, and A is the cross-sectional area of the conductor. Since it is assumed that each atom supplies one free electron, n is the same as the number of atoms per unit volume.

This may be found from $$n = \frac{\text{mass per unit volume}}{\text{mass per atom}} = \frac{\text{density}}{\text{mass per atom}}$$

The mass of a single atom is $$m_{\text{atom}} = \frac{\text{mass per mole}}{\text{atoms per mole}} = \frac{\text{molecular weight}}{\text{Avogadro's number}}$$

For aluminum, this gives $$m_{\text{atom}} = \frac{27\ \text{g}}{6.02\times10^{23}} = 4.5\times10^{-23}\ \text{g}$$

The density of free electrons is $$n = \frac{\rho}{m_{\text{atom}}} = \frac{2.7\ \text{g}/\text{cm}^3}{4.5\times10^{-23}\ \text{g}} = 6.0\times10^{22}\ \text{cm}^{-3}$$

or $$n = \left(\frac{6.0\times10^{22}}{\text{cm}^3}\right)\left(\frac{10^6\,\text{cm}^3}{1\ \text{m}^3}\right) = 6.0\times10^{28}\ \text{m}^{-3}$$

The drift speed of the electrons in this wire is then $v_d = I/neA$:

$$v_d = \frac{5.0\ \text{C}/\text{s}}{\left(6.0\times10^{28}\ \text{m}^{-3}\right)\left(1.6\times10^{-19}\ \text{C}\right)\left(4.0\times10^{-6}\ \text{m}^2\right)} = 1.3\times10^{-4}\ \text{m}/\text{s}$$ ◊