47. The heating coil of a hot water heater has a resistance of 20 Ω and operates at 210 V. If electrical energy costs \$0.080/kWh, what does it cost to raise the 200 kg of water in the tank from 15 °C to 80 °C? (See Chapter 11.)

Solution

The kilowatt-hour is a measure of energy equal to

$$1\ \text{kWh} = (1.00\ \text{kW})(1.00\ \text{h}) = (1000\ \text{J/s})(3600\ \text{s}) = 3.60\times10^6\ \text{J}$$

The energy needed to raise the temperature of 200 kg of water from 15 °C to 80 °C is

$$Q = mc_{\text{water}}(\Delta T) = (200\ \text{kg})(4186\ \text{J/kg}\cdot\text{°C})(65\ \text{°C}) = 5.4\times10^7\ \text{J}$$

In units of kWh, $$Q = (5.4\times10^7\ \text{J})\left(\frac{1.00\ \text{kWh}}{3.60\times10^6\ \text{J}}\right) = 15\ \text{kWh}$$

and the cost of operating the heater to produce this quantity of thermal energy is

$$\text{cost} = (\text{energy used})(\text{rate}) = (15\ \text{kWh})(\$0.080/\text{kWh}) = \$1.20 \qquad \Diamond$$

Observe that this cost is independent of both the resistance of the heating coil and the applied voltage.