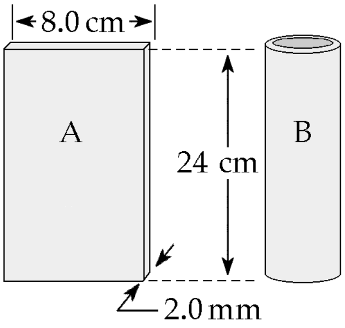
65. (a) A sheet of copper ($\rho = 1.7 \times 10^{-8}\ \Omega\cdot\text{m}$) is 2.0 mm thick and has surface dimensions of 8.0 cm × 24 cm. If the long edges are joined to form a tube 24 cm in length, what is the resistance between the ends? (b) What mass of copper is required to manufacture a 1500-m-long spool of copper cable with a total resistance of 4.5 Ω?



Solution

(a) When the sheet of copper, as shown in part (A) of the figure, is joined along the 24-cm edges, it forms a hollow cylindrical shell as shown in (B). The shell has a circumference of 8.0 cm, length of 24 cm and a thickness of 2.0 mm. The cross-sectional area of the material in the shell is the same as the area of the upper end of the sheet.

$$A = \left(2.0 \times 10^{-3}\ \text{m}\right)\left(8.0 \times 10^{-2}\ \text{m}\right) = 1.6 \times 10^{-4}\ \text{m}^2$$

The resistivity of copper is $\rho = 1.7 \times 10^{-8}\ \Omega\cdot\text{m}$, so the resistance between the ends of the shell is

$$R = \frac{\rho L}{A} = \left(1.7 \times 10^{-8}\ \Omega\cdot\text{m}\right)\left(\frac{24 \times 10^{-2}\ \text{m}}{1.6 \times 10^{-4}\ \text{m}^2}\right) = 2.6 \times 10^{-5}\ \Omega$$ ◊

(b) A solid cylindrical copper cable is to be 1500 m long and have a resistance of 4.5 Ω. The volume of copper needed will be $V = AL$ where A is the cross sectional area of the cable and $L = 1500$ m is the length.

Thus, the area may be written as $A = V/L$

and the resistance, $R = \rho L/A$, becomes $R = \dfrac{\rho L^2}{V}$

The mass of copper in the cable is $m = (\text{volume})(\text{density})$

Thus, the volume is $V = \dfrac{m}{\text{density}}$

and the resistance may be written as $R = \rho L^2\left(\dfrac{\text{density}}{m}\right)$

The density of copper is $8.92 \times 10^3\ \text{kg/m}^3$ (see Table 9.2 in the textbook) and the mass of copper needed to make the specified cable is:

$$m = \rho L^2\left(\frac{\text{density}}{R}\right) = \left(1.7 \times 10^{-8}\ \Omega\cdot\text{m}\right)(1500\ \text{m})^2\left(\frac{8.92 \times 10^3\ \text{kg/m}^3}{4.5\ \Omega}\right) = 76\ \text{kg}$$ ◊