7. What velocity would a proton need to circle Earth 1000 km above the magnetic equator, where Earth's magnetic field is directed horizontally north and has a magnitude of 4.00×10^{-8} T?

Solution

The gravitational force acting on the proton will be negligible in comparison to the magnetic force. Thus, the magnetic force must supply the needed centripetal acceleration of the proton. To do this, the magnetic force must be directed downward, toward Earth, and have a magnitude given by

$$F = qvB\sin\theta = \frac{mv^2}{r}$$

Here, q is the charge of the proton, v is its speed, θ is the angle the velocity makes with magnetic field **B**, m is the mass of the proton , and the radius of the proton's orbit is

$$r = R_E + 1000 \text{ km} = 6.38 \times 10^6 \text{ m} + 1000 \times 10^3 \text{ m} = 7.38 \times 10^6 \text{ m}$$

Since the proton moves parallel to the equator, and hence perpendicular to the south-to-north direction of the magnetic field, $\theta = 90.0°$ and the required speed is

$$v = \frac{qrB}{m} = \frac{\left(1.60 \times 10^{-19} \text{ C}\right)\left(7.38 \times 10^6 \text{ m}\right)\left(4.00 \times 10^{-8} \text{ T}\right)}{1.67 \times 10^{-27} \text{ kg}} = 2.83 \times 10^7 \text{ m/s} \quad \Diamond$$

Note that the proton must move east to west around its orbit if the magnetic force is to be directed downward.