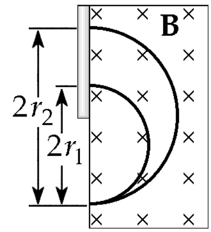
55. Two species of singly charged positive ions of masses 20.0×10^{-27} kg and 23.4×10^{-27} kg enter a magnetic field at the same location with a speed of 1.00×10^{5} m/s. If the strength of the field is 0.200 T, and the ions move perpendicularly to the field, find their distance of separation after they complete one half of their circular path.

Solution



When a charged particle moves perpendicularly to a magnetic field, a magnetic force directed perpendicular to the velocity of the particle acts on it. This force, of magnitude $F = qvB$, produces a centripetal acceleration and causes the particle to follow a circular path.

The force required to make a particle with mass m and speed v move in a circle of radius r is given by $\quad F = ma_c = mv^2/r$

Thus, $qvB = mv^2/r$, and the radius of the path is $\quad r = mv/qB$

Both species of ions are positive and singly charged $\quad (q = +e)$

and both move at a speed of $\quad v = 1.00 \times 10^{5}\ \text{m/s}$

The radii of the paths of the two types of ions are $\quad r_1 = \dfrac{m_1 v}{qB}$ and $r_2 = \dfrac{m_2 v}{qB}$

After completing one half of their circular paths, the two ions are separated by a distance equal to the difference in the diameters of their paths (see the sketch).

Thus,
$$\Delta d = 2(r_2 - r_1) = \frac{2v}{qB}(m_2 - m_1)$$

$$\Delta d = \frac{2(1.00 \times 10^{5}\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(0.200\ \text{T})}\left[(23.4 - 20.0) \times 10^{-27}\ \text{kg}\right]$$

or
$$\Delta d = 2.13 \times 10^{-2}\ \text{m} = 2.13\ \text{cm}$$ ◊